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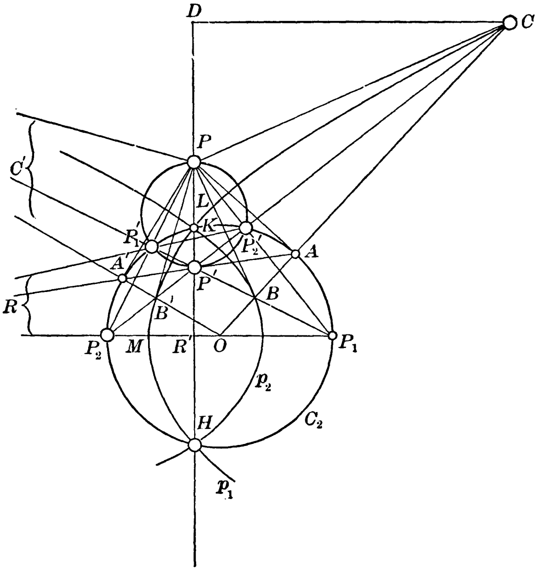
SOME PROPERTIES OF A STRAIGHT LINE AND CIRCLE AND THEIR ASSOCIATED PARABOLAS.

By James H. Weaver.

In the following article the author sets up a configuration involving circles and confocal parabolas and proves some theorems which he believes to be new.

Let there be a straight line l and a circle C_2 with center O and radius r. The locus of the center of a circle tangent to C_2 and l is two parabolas, with the common focus O, and a common axis, which is the line through O perpendicular to l, and with lines parallel to l and at a distance r from l as directrices. These parabolas may be said to be associated with l and C_2. Call these parabolas p_1 and p_2. Conversely, with any two parabolas having a common axis and focus, there are associated a straight line l and a circle, C_2.



Theorem I: *In the configuration of a straight line l and a circle C_2 let tangents be drawn from any point P on l to C_2 and to the associated parabolas p_1 and p_2. The points of contact of these tangents lie on two straight lines passing through O.*

Proof: Let the points of contact (Fig.) of the tangents from P to C_2 be A and A', of those from P to p_2 be B and C', and of those from P to p_1 be C and B'. Then A, B, and C are collinear with O, and A', B' and C' are collinear with O. For suppose PA drawn from P tangent to C_2. Then draw OA and extend it until it cuts p_1 in some point C. Draw CD perpendicular to l and then draw CP. Since C is on p_1 and CP bisects $\measuredangle DCO$, PC is tangent to p_1 at C. A similar proof holds for the other points of contact.

Theorem II: *The polars of P with respect to p_1 and p_2 intersect C_2 in four points, which determine a quadrangle, whose diagonal points are P, the point P' on l, which is conjugate to P with respect to C_2, and the pole of l with respect to C_2.*

Proof: Let vertex of p_1 be M. Then since $P_2M = MR'$, P_2 is pole of l with respect to p_1. It follows analytically that P_1P_1' and P_2P are perpendicular, and a circle on PP' as diameter passes through P_1' and P_2' and cuts C_2 orthogonally. For if L is the center of the circle on PP', then $\measuredangle LP_2'O$ is right. Because $\measuredangle OP_2'P_2 = \measuredangle OP_2P_2'$ and $\measuredangle LP_2'P_2 = \measuredangle LP'P_2' = \measuredangle P_2P'R'$. But $\measuredangle OP_2P' + \measuredangle P_2P'R' =$ rt $\measuredangle$, $\therefore \measuredangle LP_2'O$ is right. And tangents to C_2 at P_1' and P_2' will intersect on l. $\therefore$ the pole of $P_1'P_2'$ lies on l. It is also evident that the pole of P_1P_2 is the point at infinity on l. Let P_1P_2 and $P_1'P_2'$ intersect in R. Then R is the pole of l with respect to C_2.

The circle on PP' as diameter is one of a system of limit point circles, which have P_1P_2 for radical axis; and R is the radical center of C_2 and any two circles of the system.

From the above discussion the following is evident.

Theorem III: *The perpendiculars from P to the polars of P with respect to p_1 and p_2 intersect these polars on C_2.*

OHIO STATE UNIVERSITY.